CIS 66 **Final Exam, Spring 2000**

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Name:

SSN:

Section:

This exam consists of 11 questions.
Good Luck.

Question	**Points**	**Out of**
1		12
2		12
3		12
4		12
5		12
6		12
7		12
8		12
9		12
10		12
11		12
Best 10		120

1 Answer the following.

(a) If $1 + 1 = 2$, then 1 is a prime. `true` `false`

(b) Paris is in France and $2 + 2 = 5$. `true` `false`

(c) $(\forall$ integer $n)(\exists$ integer $m)(n^2 - m^2 = 9)$. `true` `false`

(d) $p \rightarrow p \vee q$, `tautology` `contradiction` `contingency`

(e) $\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$, `tautology` `contradiction` `contingency`

(f) Are $\neg\exists x \forall y P(x, y)$ and $\forall x \neg(\forall y P(x, y))$ equivalent? `true` `false`

2 In each of the following, f is a function from set S into S. Circle the properties of f.

(a) $S = \{1, 2, 3, 4, 5, 6, 7\}, \ f(n) = 8 - n.$ `1-1` `onto` `both` neither

(b) $S = \{1, 2, 3, 4, 5, 6\}, \ f(n) = (n^2 \bmod 6) + 1.$ `1-1` `onto` `both` neither

(c) $S = \{1, 2, 3, \ldots\}, \ f(n) = \lceil \frac{n}{2} \rceil.$ `1-1` `onto` `both` neither

Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+1}$.

(d) Evaluate $(f \circ g)(3)$

(e) Evaluate $(g \circ f)(-3)$

(f) Let $f(n)$ be the number messages consisting of n bits, where each message is made up from the bitstrings **01** and **011**. Find a recurrence relation with initial conditions for $f(n)$.

3 Make a truth table for the propositional form $[(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R)] \rightarrow R$

4 Let $U = \{1, 2, \ldots, 12\}$, $X = \{2, 3, 5, 7, 11\}$, $A = \{1, 2, 4, 8\}$, $C = \{1, 2, 5, 7, 9\}$

(a) $\emptyset \in X$. `true` `false`

(b) $(A \cap C) \subseteq X$. `true` `false`

(c) List all the elements of $S = \{x \in U | x \text{ is a positive odd integer and } x < 9\}$

(d) Form $(A \cup S)'$.

Assume $|A| = 50$, $|B| = 50$, $|C| = 60$, $|A \cap B| = 28$, $|B \cap C| = 17$, $|A \cap C| = 12$, and $|A \cap B \cap C| = 9$.

(e) Find $|A \cup B \cup C|$

(f) Find $|A \cap B \cap \overline{C}|$.

5 Find the least n such that $f(x)$ is $O(x^n)$.

(a) $f(x) = 4x^3 + 100x \log x$.

(b) $f(x) = (4x^3 + 100x \log x)/(x^2 - 1)$.

Give as good a big$-O$ estimate as possible for each of the following functions.

(c) $(n\, log\, n + n^2)(n^3 + 2)$.

(d) $(n\sqrt{n} + 10)^2 + (\log n + 1)(n^2 + 100n)$.

6 Answer the following.

(a) Give a prime factorization of 306.

(b) Evaluate 144 mod 11

(c) Evaluate $\sum_{j=0}^{6} 2 \cdot (-3)^j$

Find the ones complement, using bitstrings of length seven, of the following integers.

(d) 44

(e) -21

Convert

(f) 111 from decimal to binary.

7 Using the Principle of Mathematical Induction, show that

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n - 1)}{2}, \; n \geq 1$$

8 (a) What is the probability that a 5-card poker hand contains exactly 2 Kings?

(b) What is the probability that a 5-card poker hand contains 2 Kings and another pair?

How many bitstrings of length 9 have

(b) at least two 1's?

(c) exactly 6 1's if each 0 must be followed by a 1?

9 Solve the following recurrence relations.

(a) $a_n = 7a_{n-1} - 10a_{n-2}, n \geq 2$ with inititial condition $a_0 = 2,\ a_1 = -1$

(b) $a_n = 6a_{n-1} - 9a_{n-2}, n \geq 2$ with inititial condition $a_0 = 2,\ a_1 = -1$

10 Let $A = \{2, 3, 4, 6, 9, 11$ Let R be the relation on A defined by $(a, b) \in R$ if, and only if, a and b have a common prime divisor.

(a) Draw the directed graph (digraph) corresponding to the relation.

(b) Circle the properties R possesses.

reflexive symmetric antisymmetric transitive

(c) List the pairs $(a, b) \in R^2$ with $a \neq b$.

(d) By listing **additional** ordered pairs , find the **smallest** relation containing the relation R that is an equivalence relation.

11 Let the relation R be represented by the matrix

$$M_R = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

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(a) Draw the directed graph (digraph) corresponding to the relation.

(b) Find M_{R^2}.

(c) Find $B = M_R \vee M_{R^2}$.

(d) Draw the directed graph (digraph) for B.